Transport in Porous Media (2021) 139:527–558
https://doi.org/10.1007/s11242-021-01678-z

Recursive Mixed Multiscale Model Reduction for Karst Conduit-Flow in Carbonate Reservoirs

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Received: 6 April 2021 / Accepted: 17 August 2021 / Published online: 16 September 2021

Abstract

We devise a new computational model to describe numerically the coarse-scale response of coupled conduit/matrix flow in karstified carbonate rocks. The methodology is based on the combination of a two-scale coupled 1D/3D flow model, rigorously derived by formal homogenization, and an extension of the recursive Mixed Multiscale Method (MuMM), seated on domain decomposition and redesigned herein to more complex bi-modal systems. Within this double-coarsening framework, our target macroscopic scenario consists of a limestone matrix intertwined by a network of karst-conduits of high aspect ratio between longitudinal length and hydraulic diameter. In order to upscale the high-fidelity flow equations to a mesoscale model characterized by the presence of 1D sub-manifolds, associated with the conduit network, embedded in the 3D carbonate matrix, we proceed within a model reduction seated on matched asymptotic expansions. Such an upscaling leads to the appearance of an exchange function described by a line-source δ-function localized in the coordinates intercepted by the conduit network. Subsequently, the extended multiscale method is constructed to numerically capture the macroscopic response of the karstified medium by properly designing multiscale basis functions for the mixed formulation of the reduced problem, adapted to the scenario with presence of the local conduit/matrix exchange functions, numerically approximated by a local regularization of the δ-function. Within the framework of the recursive multiscale domain decomposition methodology, the coarse-grid interface problem, which assembles the local solutions, is replaced by the family of small localized interface linear systems obtained by the clustering of the multiscale basis functions associated with the nearest neighbor subdomains. Parallel implementation of the recursive procedure proposed herein exhibits good efficiency and scalability properties, showing great potential for the resolution of giant karstified reservoirs containing up to billions of cells. Computational simulations are performed considering a particular karst arrangement of branchwork-type in the sense of the pioneering work of Palmer (Geol Soc Am Bull 103: 1–21, 1991). To assess the quality of the coarser-scale multiscale solutions, numerical results of pressure and velocity fields are computed and compared with a reference solution computed by the mixed-hybrid finite element method adopting fine meshes.

Article Hsighlights

- A reduced 1D/3D coupled model is constructed within the framework of formal homogenizationfor flow in karst conduits.

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